Extrêmes

- Gilles Stupfler (Université Angers)

  Title: Inference for extremal regression with dependent heavy-tailed data

  Abstract: Nonparametric inference on tail conditional quantiles and their least squares analogs, expectiles, remains limited to i.i.d. data. We develop a fully operational inferential theory for extreme conditional quantiles and expectiles in the challenging framework of α-mixing, conditional heavy-tailed data whose tail index may vary with covariate values. This requires a dedicated treatment to deal with data sparsity in the far tail of the response, in addition to handling difficulties inherent to mixing, smoothing and sparsity associated with covariate localization. We prove the pointwise asymptotic normality of our estimators and obtain optimal rates of convergence reminiscent of those found in the i.i.d. regression setting, but which had not been established in the conditional extreme value literature. Our assumptions hold in a wide range of models. We propose full bias and variance reduction procedures, and simple but effective data-based rules for selecting tuning hyperparameters. Our inference strategy is shown to perform well in finite samples and is showcased in applications to stock returns and tornado loss data. This is joint work with Abdelaati Daouia and Antoine Usseglio-Carleve.

- Jonathan El Methni (Université Grenoble Alpes)

  Title: Reduced-bias estimation of extreme risk measures for heavy-tailed distributions

  Abstract: Conditional tail expectation (CTE) is a coherent risk measure defined as the mean of the loss distribution above a high quantile.The existence of the CTE as well as the asymptotic properties of associated estimators however require integrability conditions that may be violated when dealing with heavy-tailed distributions. We introduce Box-Cox transforms of the CTE that have two benefits. First, they alleviate these theoretical issues. Second, they enable the recovery of a number of risk measures such as conditional tail moments. The construction of dedicated estimators is based on the investigation of the asymptotic relationship between Box-Cox transforms of the CTE and quantiles at extreme probability levels, as well as on an extrapolation formula established in the heavy-tailed context. We quantify and estimate the bias induced by the use of these approximations and then introduce reduced-bias estimators whose asymptotic properties are rigorously shown. Their finite-sample properties are assessed on a simulation study and illustrated on real data, highlighting the practical interest of both the bias reduction and the Box-Cox transform. This is joint work with Michaël Allouche and Stéphane Girard.

- Juliette Legrand (Université Brest)

  Title: Tail-related risk measures estimation using stochastic simulation of multivariate extremes

Abstract: This study focuses on the estimation of risk measures at extreme levels, mainly the expected shortfall (ES) and the mean expected shortfall. For a given level α, the ES of a random variable X, is defined as $\mathbb{E}[X|X > \mathrm{VaR}_\alpha(X)]$, where $\mathrm{VaR}_\alpha(X)$ denotes the value at risk. In order to estimate this quantity accurately, our approach consists of increasing the number of extreme samples available above the VaR level. To do so, we rely on a non-parametric simulation approach proposed in a previous study, which allows to simulate jointly and conditionally asymptotically dependent extremes, based on a specific characterisation of multivariate extremes with multivariate generalised Pareto distributions. Numerical study shows the true benefit of employing this procedure in terms of the estimates accuracy, over more sophisticated existing approaches. This is joint work with Nisrine Madhar and Maud Thomas.

- Antoine Usseglio-Carleve (Université Avignon)

  Title: ANalysis Of Variability for heavy-tailed EXtremes

  Abstract: Analysis of variance (ANOVA) is commonly employed to assess differences in the means of independent samples. However, it is unsuitable for evaluating differences in tail behavior, especially when means do not exist or empirical estimation of moments is inconsistent due to heavy-tailed distributions. Here, we propose an ANOVA-like decomposition to analyze tail variability, allowing for flexible representation of heavy tails through a set of user-defined extreme quantiles, possibly located outside the range of observations. Building on the assumption of regular variation, we introduce a test for significant tail differences among multiple independent samples and derive its asymptotic distribution. We investigate the theoretical behavior of the test statistics for the case of two samples, each following a Pareto distribution, and explore strategies for setting hyperparameters in the test procedure. The finite-sample performance in simulations reveals generally reliable test behavior for a wide range of situations. The test is applied to identify clusters of financial stock indices with similar extreme log-returns and to detect temporal changes in daily precipitation extremes at rain gauges in Germany. This is joint work with Stéphane Girard and Thomas Opitz.